Application of dynamic tilt correction with direct measurements of rotation

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Motivation

Dynamic tilt correction is one of many applications to be applied to real 6DoF field observations.

Critical for:
- long period signals
- large rotation angles

Volcano seismology

Ocean bottom seismology

Engineering / strong motion
Dynamic tilt - A well known problem for seismology

\[ \ddot{y} + 2\omega_0\zeta \dot{y} + \omega_0^2 y = -\ddot{U}_Y + \Theta_X g \]  \hspace{1cm} (1)

- \( y \) displacement of seismometer mass in Y-direction
- \( U_Y \) ground displacement in Y-direction
- \( \omega_0, \zeta \) natural frequency and fraction of critical damping
- \( \Theta_X \) rotation angle around X-axis
- \( g \) gravitational acceleration

adapted from Trifunac et al. 2001
Application example

- Measurement campaign on Mt. Stromboli, Italy
- September 2018
- 3 6C stations
  - translation: 3C broadband seismometers Trillium Compact
  - rotation: 3C rotational seismometers BlueSeis3A
Application example

- example event from station TOR2
- distance source - receiver $\approx 200$ m
How to correct for dynamic tilt? - Time domain

for the Y-component of the seismometer:

\[ \ddot{y} + 2\omega_0 \zeta \dot{y} + \omega_0^2 y = -\ddot{U}_Y + \Theta_X g \]

add the rotation angle around the Y-axis times g to the seismometer output in acceleration sample by sample
How to correct for dynamic tilt? - Frequency domain (plain)

- $s(t)$: source of the disturbance, **tilt angle recording**
  - $r(t)$: response to the disturbance, **acceleration recording**
    - $S(f)$: Fourier transform of $s(t)$
    - $R(f)$: Fourier transform of $r(t)$
    - $G_{ss}(f)$: autospectral density of source
    - $G_{rr}(f)$: autospectral density of receiver
- coherency between source and response $\gamma_{rs}(f) \Rightarrow$ transfer function $A_{rs}(f) = \gamma_{rs}(f) \cdot \sqrt{\frac{G_{rr}(f)}{G_{ss}(f)}}$
- corrected response: $R'(f) = R(f) - A_{rs}^* S(f)$

after Crawford and Webb (2000), BSSA
How to correct for dynamic tilt? - Frequency domain (adapted)

- $s(t)$: source of the disturbance, **tilt angle recording**
  - $r(t)$: response to the disturbance, **acceleration recording**
  - $S(f)$: Fourier transform of $s(t)$
  - $R(f)$: Fourier transform of $r(t)$
  - $G_{ss}(f)$: autospectral density of source
  - $G_{rr}(f)$: autospectral density of receiver
- coherency between source and response $\gamma_{rs}(f)$
- only parts of the spectra with significant coherency are used ($> 0.5$)

$$|S(f)| = \begin{cases} 
|S(f)| & \text{if } |\gamma_{rs}(f)| > 0.5 \\
0 & \text{otherwise}
\end{cases}$$

- corrected response: $R'(f) = R(f) - 9.81 \cdot S(f)$

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Application example

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Questions

Where do the differences come from?

Which one is the best method?
Answers

Step table experiment

• reproducible laboratory experiment
• well known input

Synthetic experiment

• simulation of a VLP event in a volcanic environment
• more realistic signal
• known input and output
The step-table experiment

rotation around Y-axis introduces acceleration in X-direction!

- $\Theta_X g$ "classical" tilt contribution
- $\ddot{U}_Y^{\text{add}}$ after correction: residual displacement
Step-table experiment - Results

Application of dynamic tilt correction with direct measurements of rotation
Application of dynamic tilt correction with direct measurements of rotation
Step-table experiment - Results in detail (adapted)

Application of dynamic tilt correction with direct measurements of rotation
Synthetic experiment - Gaussian hill geometry

Application of dynamic tilt correction with direct measurements of rotation
Synthetic experiment - The source

gaussian source-time function with:

\[
M = \begin{bmatrix}
3m_0 & 0 & 0 \\
0 & m_0 & 0 \\
0 & 0 & m_0
\end{bmatrix}
\]

\[m_0 = 1.27 \cdot 10^{13} \text{ Nm}\]

Simulated 3d seismic wave propagation with SW4 2.0*

Focus on

- acceleration in X-direction
- rotation around Y-axis

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Synthetic experiment - Results (10 s - 30 s)
The time domain problem

Self-noise level of BS3A

- time domain correction adds a lot of instrument noise at low frequencies!
Conclusion / Outlook

• Dynamic tilt correction is a critical step in data analysis
• Difference in peak displacement up to 20% in our example
• Time domain method adds significant noise to seismometer data

For the future:

• not only single events but continuous data processing
• more data from

Volcano seismology  Ocean bottom seismology  Engineering / strong motion
Thank You!

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ROtational Motions in seismology